CP-Conserving Unparticle Phase Effects on the Unpolarized and Polarized Direct CP Asymmetry in $b \to d\ell^+\ell^-$ Transition

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Abstract

We examine the unparticle CP-conserving phase effects on the direct CP asymmetry for both polarized and unpolarized lepton in the inclusive $b \to d\ell^+\ell^-$ transition, where the flavor changing neutral currents are forbidden at tree level but are induced by one-loop penguin diagrams. The averaged polarized and unpolarized CP asymmetries depict strong dependency on the unparticle parameters. In particular, a sizable discrepancy corresponding to the standard model is achieved when the scale dimension value is $1 < d_{\mathcal{U}} < 2$. We see that the unparticle stuff significantly enhances, suppresses or changes the sign of the CP asymmetry depending on the definite value of the scaling dimension $d_{\mathcal{U}}$. Especially, when $d_{\mathcal{U}} \sim 1.1$ the CP asymmetries vanish.

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1 Introduction

Georgi [1, 2] has recently proposed that unparticles stuff which can couple to the standard model (SM) particles at the Tev Scale. Unparticles are massless and invisible coming out of a scale invariant sector with non-integer scaling dimension $d_{\mathcal{U}}$ when decoupled at a large scale. The propagator of these invisible unparticles includes CP conserving phase which is dependent on the non-integer scaling dimension $d_{\mathcal{U}}$ [2]. The virtual unparticle propagation and its effects were first studied by Georgi himself[2]. Moreover, the CP conserving phase of the unparticles and its effects in FCNC, especially, in hadronic and semileptonic B decays have been studied in [3, 4, 5, 6].

A phenomenological study needs construction of the effective Hamiltonian to describe the interactions of unparticles with the SM fields in the low energy level[7]. So that, we can investigate the effects of the possible scale invariant sector, experimentally.

The direct search of the unparticles is based on the study of missing energies at various processes which can be measured at LHC or ILC. The indirect search includes the dipole moments of fundamental particles, lepton flavor violation(LFV) and flavor changing neutral current(FCNC) processes where the virtual unparticles enter as mediator. Note that, the phenomenological studies considering the direct and indirect search on unparticles have been progressing [2]-[14]; their effects on the missing energy of many processes; the anomalous magnetic moments; the electric dipole moments; $D^0 - \bar{D}^0$ and $B^0 - \bar{B}^0$ mixing; lepton flavor violating interactions; direct CP violation in particle physics; the phenomenological implications in cosmology and in astrophysics.

It is well known that in a decay process the existence of direct CPA(A_{CP}) requires: firstly at least two different terms in decay amplitude. Secondly, These terms must depend on two types of phases named weak(δ) and strong(ϕ) phases. The weak phase is CP violating and strong phase is CP conserving phase. The A_{CP} depends on the interference of different amplitude and is proportional to the phases. i.e.,

$$A_{CP} \propto \sin(\delta) \sin(\phi) \tag{1}$$

The sizable value of A_{CP} can be obtained if both phases are non-zero and large. The weak phase of the SM is a unique phase of the Cabibbo-Kobayashi-Maskawa(CKM) quark mixing matrix. This weak phase is a free parameter of the SM and can not be fixed by SM itself but it has been fixed by experimental methods[15]. Unlike the weak Phase, the CP conserving strong phase is process dependent(not unique). The theoretical calculation of the strong phase

is in general hard due to the hadronic uncertainty. The CP conserving unparticle phase exist in the propagators beside the strong phase can affect the value of the A_{CP} in some decay processes(see Eq. 1). To explore this possibility, Chuan-Hung Chen, *et al.* concentrated on some pure hadronic and pure leptonic B decays[3, 8].

We aim to study the possible effects of the CP conserving phase in semileptonic B decays. Rare semileptonic decays $b \to s(d)\ell^+\ell^-$ are more informative for this aim, since these decays are relatively clean compared to pure hadronic decays. It is well known that the matrix element for the $b \to s\ell^+\ell^-$ transition involves only one independent CKM matrix element, namely $|V_{tb}V_{ts}^*|$, so the CP-violation in this channel is strongly suppressed in the SM considering the above mentioned requirements of the CPA which requires the weak phase. However, the possibility of CP-violation as a result of the new weak phase coming out of the physics beyond the standard model in $b \to s$ transition has been studied in supersymmetry[16]-[17], fourth-generation standard model(SM4)[18]-[21] and minimal extension of the SM[22]. Situation for $b \to d\ell^+\ell^-$ is totaly different from $b \to s\ell^+\ell^-$ transition. In this case, all CKM matrix elements $|V_{td}V_{tb}^*|$, $|V_{cd}V_{cb}^*|$ and $|V_{ud}V_{ub}^*|$ are in the same order and for this reason the matrix element of $b \to d\ell^+\ell^-$ transition contains two different amplitudes with two different CKM elements and therefore sizable CPA is expected[23, 24]. Here, we study the effects of the CP conserving unparticle phase on CP asymmetry in the $b \to d\ell^+\ell^-$ transition with unpolarized and polarized lepton cases.

This study encompasses four sections: In Section 2, we present the the effective lagrangian and effective vertices which drive the FCNC decays with vector unparticle mediation. In section 3, we calculate the polarized and unpolarized CP asymmetries. Section 4 is devoted to the discussion and our conclusions.

2 Flavor changing neutral currents mediated by vector unparticle

The starting point of the idea is the interaction between two sectors, the SM and the ultraviolet sector with non-trivial infrared fixed point, at high energy level. The ultraviolet sector appears as new degrees of freedom, called unparticles, being massless and having non integral scaling dimension $d_{\mathcal{U}}$ around, $\Lambda_{\mathcal{U}} \sim 1\,TeV$. This mechanism results in the existence of the effective field theory with effective Lagrangian in the low energy level. One may for simplicity assume that

unparticles only couple to the flavor conserving fermion currents[8], described by [1, 2, 10, 11]

$$\frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{f}\gamma_\mu \left(C_L^{\mathrm{f}} P_L + C_R^{\mathrm{f}} P_R \right) f O_{\mathcal{U}}^\mu \tag{2}$$

where $O_{\mathcal{U}}^\mu$ is the unparticle operator. Similar to the SM, FCNCs such as $f \to f'\mathcal{U}$ can be induced by the charged weak currents at quantum loop level and clearly, neutral current $f \to f\mathcal{U}$ is flavor diagonal.
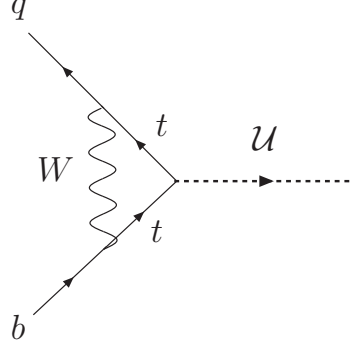

Figure 1: Feynman diagram for $b \to q(s \text{ or } d)\mathcal{U}$, where t is top quark.

The leading order of effective Hamiltonian for the Fig. 1 can be written as follows:

$$\mathcal{L}_{\mathcal{U}} = \frac{g^2}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} V_{tb} V_{tq}^* C_L^{qb} \bar{q}\gamma_\mu P_L b \, O_{\mathcal{U}}^\mu \,, \tag{3}$$

where

$$C_L^{qb} = \frac{1}{(4\pi)^2} I(x_t)\,,$$
$$I(x_t) = \frac{x_t(2C_R^t + C_L^t x_t)}{2(1-x_t)^2}\left(-1 + x_t - \ln x_t\right)\,, \tag{4}$$

with $x_t = m_t^2/m_W^2$[8].

To obtain the effective Hamiltonian for $b \to qf\bar{f}$ transition where unparticles enter as mediators, we must obtain the unparticle propagator, which is given by [1, 2, 10, 11]

$$\int d^4 x e^{ip \cdot x} \langle 0|T\left(O_{\mathcal{U}}^\mu(x) O_{\mathcal{U}}^\nu(0)\right)|0\rangle = i\Delta_{\mathcal{U}}(p^2)\, e^{-i\phi_{\mathcal{U}}} \tag{5}$$

where

$$\Delta_{\mathcal{U}}(p^2) = \frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)} \frac{-g^{\mu\nu} + ap^\mu p^\nu/p^2}{(p^2+i)^{2-d_{\mathcal{U}}}}\,,$$
$$\phi_{\mathcal{U}} = (d_{\mathcal{U}} - 2)\pi\,, \tag{6}$$

3

where $a = 1$ for transverse $O_\mathcal{U}^\mu$ and $a = \frac{2(d-2)}{d-1}$ in the conformal field theories(CFT)[12]. Note also that, the contribution from the longitudinal piece $ap^\mu p^\nu/p^2$ in Eq. (6) can be dropped for massless or light external fermions. In this case, Georgi[2] and Grinstein $et.\ al,$[12] approaches provide the same result. Also,

$$A_{d_\mathcal{U}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_\mathcal{U}}} \frac{\Gamma(d_\mathcal{U} + 1/2)}{\Gamma(d_\mathcal{U} - 1)\Gamma(2d_\mathcal{U})}\,. \tag{7}$$

Note that, in Eq. (5) the phase factor arises from $(-1)^{d_\mathcal{U}-2} = e^{-i\pi(d_\mathcal{U}-2)}$ and, here, the massless vector unparticle operator is conserved current, i.e., $\partial_\mu O_\mathcal{U}^\mu = 0$. The effective Hamiltonian for $b \to q f \bar{f}$ just with the contribution of the vector unparticle as a mediator can be given as:

$$\mathcal{H}_\mathcal{U} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{tq}^* \tilde{\Delta}_\mathcal{U}(p^2) e^{-i\phi_\mathcal{U}}\, \bar{q}\gamma_\mu P_L b\, \bar{f}\,\gamma^\mu \left(C_L^f P_L + C_R^f P_R\right) f\,, \tag{8}$$

where

$$\tilde{\Delta}_\mathcal{U}(p^2) = 8C_L^{qb} \frac{A_{d_\mathcal{U}}}{2\sin d_\mathcal{U}\pi} \frac{m_W^2}{p^2} \left(\frac{p^2}{\Lambda_\mathcal{U}^2}\right)^{d_\mathcal{U}-1}\,. \tag{9}$$

Here, f stands for fermions. i.e., f can be neutrinos or charged leptons or quarks.

3 $\quad b \to d\ell^+\ell^-$ transition in the presence of the vector unparticle as a mediator

By the extension of the $b \to d\mathcal{U}$ to study the semileptonic decays of $b \to d\ell^+\ell^-$, the decay amplitude in the presence of the vector unparticle as a mediator can be obtained. Here, again we assume that unparticle coupled to the leptons are flavor conserving. The penguin diagram describing this decay is shown in Fig. 2. Due to the CKM suppression, the semileptonic decays with $b \to d$ are much less than those of $b \to s$. However, it is worth to study the $b \to d$ transition beyond the $b \to s$ one because the CKM matrix element V_{td} carries a CP violating weak phase, which is almost vanishes in the $b \to s$ transition. Thus, $b \to d\ell^+\ell^-$ decay could be even more interesting on CP violation in the framework of unparticle physics. we will focus on the CP violating asymmetry in $b \to d\ell^+\ell^-$.

The QCD corrected effective Lagrangian for the decays $b \to d\ell^+\ell^-$ can be obtained by integrating out the heavy quarks and the heavy electroweak bosons are reads as follows in the SM:

$$\begin{aligned}
M = \frac{G_F \alpha_{em}\lambda_t}{\sqrt{2}\pi}[\quad &C_9^{eff}(\bar{d}\gamma_\mu P_L b)\bar{\ell}\gamma_\mu\ell + C_{10}(\bar{d}\gamma_\mu P_L b)\bar{\ell}\gamma_\mu\gamma^5\ell \\
&-2\,C_7\bar{d}i\sigma_{\mu\nu}\frac{q^\nu}{q^2}(m_b P_R + m_s P_L)b\bar{\ell}\gamma_\mu\ell\,],
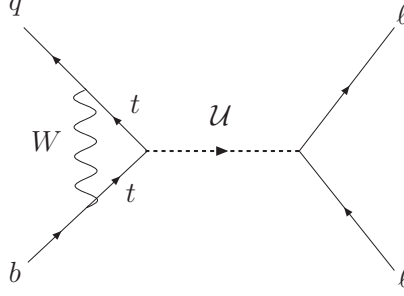\end{aligned} \tag{10}$$

4

Figure 2: $b \to q\ell^+\ell^-$ decays induced by unparticle penguin diagram.

In writing this, unitarity of the CKM matrix has been used and the term proportional to $\lambda_t = V_{tb}^* V_{td}$ has been factored out. where q denotes the four momentum of the lepton pair and C_i's are Wilson coefficients. Neglecting the terms of $O(m_q^2/m_W^2)$, $q = u, d, c$, the analytic expressions for all Wilson coefficients, except C_9^{eff}, can be found in [25]. The values of C_7^{eff} and C_{10} in leading logarithmic approximation are:

$$C_7^{eff} = -0.315, \quad C_{10} = -4.642 \tag{11}$$

only C_9^{eff} has weak and strong phases, i.e. :

$$C_9^{eff} = \xi_1 + \lambda_u \xi_2 \tag{12}$$

where the CP violating parameter λ_u is as follows:

$$\lambda_u = \frac{V_{ub}^* V_{ud}}{V_{tb}^* V_{td}} = \frac{\rho(1-\rho) - \eta^2}{(1-\rho)^2 + \eta^2} - i\frac{\eta}{(1-\rho)^2 + \eta^2} + \cdots \tag{13}$$

The explicit expressions of functions ξ_1 and ξ_2 in $\mu = m_b$ can be found in [25]–[30]: Note that, we neglect long-distance resonant contributions in C_9^{eff} for simplicity, a more complimentary and supplementary analysis of the above decay has to take the long-distance contributions, which have their origin in real intermediate $c\bar{c}$ bound states, in addition to the short-distance contribution into account.

The Wilson coefficients of the SM are modified by the introducing the vector type unparticles. It is easy to see that unparticles in this study are introduced in the way that new operators do not appear. In other words, the full operator set for the unparticle contributions are exactly the same as in the SM. The unparticle effects with the SM contributions can be derived by using $C_9^{\mathcal{U}}$ and $C_{10}^{\mathcal{U}}$, defined by

$$C_9^{\mathcal{U}}(q^2) = C_9^{\text{eff}} + \frac{\pi}{\alpha_{em}} \frac{C_R^\ell + C_L^\ell}{2} \tilde{\Delta}_{\mathcal{U}}(q^2) e^{-i\phi_{\mathcal{U}}},$$

$$C_{10}^{\mathcal{U}}(q^2) = C_{10} + \frac{\pi}{\alpha_{em}} \frac{C_R^\ell - C_L^\ell}{2} \tilde{\Delta}_{\mathcal{U}}(q^2) e^{-i\phi_{\mathcal{U}}},$$
$$C_7^{\mathcal{U}}(q^2) = C_7(q^2) \tag{14}$$

instead of C_9^{eff} and C_{10}, respectively. Where C_7 remain the same as the SM, we can rewrite $C_i^{\mathcal{U}}$'s in the m_b scale[25]. Then $C_9^{\mathcal{U}}$ will be as:

$$C_9^{\mathcal{U}} = \xi_1^{\mathcal{U}} + \lambda_u \xi_2 \tag{15}$$

where

$$\xi_1^{\mathcal{U}} = \xi_1 + \frac{\pi}{\alpha_{em}} \frac{C_R^\ell + C_L^\ell}{2} \tilde{\Delta}_{\mathcal{U}}(q^2) e^{-i\phi_{\mathcal{U}}}, \tag{16}$$

Neglecting any low energy QCD corrections ($\sim 1/m_b^2$) [32, 31] and setting the down quark mass to zero, the unpolarized differential decay width as a function of the invariant mass of the lepton pair is given by:

$$\left(\frac{d\Gamma}{d\hat{s}}\right)_0 = \frac{G_F^2 m_b^5}{192\pi^3} \frac{\alpha_{em}^2}{4\pi^2} |\lambda_t|^2 (1-\hat{s})^2 \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}} \triangle^{\mathcal{U}} \tag{17}$$

with

$$\triangle^{\mathcal{U}}(\hat{s}) = 4\frac{(2+\hat{s})}{\hat{s}}\left(1 + \frac{2\hat{m}_\ell^2}{\hat{s}}\right)|C_7^{\text{eff}}|^2 + (1+2\hat{s})\left(1 + \frac{2\hat{m}_\ell^2}{\hat{s}}\right)|C_9^{\mathcal{U}}|^2$$
$$+ (1 - 8\hat{m}_\ell^2 + 2\hat{s} + \frac{2\hat{m}_\ell^2}{\hat{s}})|C_{10}^{\mathcal{U}}|^2 + 12(1 + \frac{2\hat{m}_\ell^2}{\hat{s}})Re(C_9^{\mathcal{U}*}C_7^{\text{eff}}). \tag{18}$$

The explicit expression for the unpolarized particle decay rate $(d\Gamma/d\hat{s})_0$ has been given in (17). Obviously, it can be written as a product of a real-valued function $r(\hat{s})$ times the function $\Delta(\hat{s})$, given in (18); $(d\Gamma/d\hat{s})_0 = r(\hat{s}) \Delta(\hat{s})$. In the unpolarized case, the CP-Violating asymmetry rate can be defined by

$$A_{CP}^{\mathcal{U}}(\hat{s}) = \frac{(\frac{d\Gamma}{d\hat{s}})_0 - (\frac{d\bar{\Gamma}}{d\hat{s}})_0}{(\frac{d\Gamma}{d\hat{s}})_0 + (\frac{d\bar{\Gamma}}{d\hat{s}})_0} = \frac{\Delta^{\mathcal{U}} - \bar{\Delta}^{\mathcal{U}}}{\Delta^{\mathcal{U}} + \bar{\Delta}^{\mathcal{U}}}. \tag{19}$$

where

$$\frac{d\Gamma}{d\hat{s}} \equiv \frac{d\Gamma(b \to d\ell^+\ell^-)}{d\hat{s}}, \quad \frac{d\bar{\Gamma}}{d\hat{s}} \equiv \frac{d\bar{\Gamma}(\bar{b} \to \bar{d}\ell^+\ell^-)}{d\hat{s}} \tag{20}$$

where, $(d\bar{\Gamma}/d\hat{s})_0$ can be obtained from $(d\Gamma/d\hat{s})_0$ by making the replacement

$$C_9^{\mathcal{U}} = \xi_1^{\mathcal{U}} + \lambda_u \xi_2 \to \bar{C}_9^{\mathcal{U}} = \xi_1^{\mathcal{U}} + \lambda_u^* \xi_2. \tag{21}$$

Note that the term proportional to λ_u, CP violating parameter remains the same as the SM. Moreover, the CP violating parameter just enters into the $C_9^{\mathcal{U}}$ expression same as the SM ones. Consequently, the rate for anti-particle decay can be obtained by the following replacement in the Eq. (18):

$$\bar{\Delta}^{\mathcal{U}} = \Delta_{|\lambda_u \to \lambda_u^*}^{\mathcal{U}} \tag{22}$$

Using (19), the CP violating asymmetry is evaluated to be:

$$A_{CP}^{\mathcal{U}}(\hat{s}) = \frac{-2\,\mathrm{Im}(\lambda_u)\Sigma^{\mathcal{U}}(\hat{s})}{\Delta^{\mathcal{U}}(\hat{s}) + 2\,\mathrm{Im}(\lambda_u)\Sigma^{\mathcal{U}}(\hat{s})} \approx -2\,\mathrm{Im}(\lambda_u)\frac{\Sigma^{\mathcal{U}}(\hat{s})}{\Delta^{\mathcal{U}}(\hat{s})}. \tag{23}$$

In (23),

$$
\begin{aligned}
\Sigma^{\mathcal{U}}(\hat{s}) &= \mathrm{Im}[\xi_1^{\mathcal{U}*}\xi_2]f_+(\hat{s}) + \mathrm{Im}(C_7^{eff*}\xi_2)f_1(\hat{s}) \\
f_+(\hat{s}) &= (1+2\hat{s})\left(1 + \frac{2\hat{m}_\ell^2}{\hat{s}}\right) \\
f_1(\hat{s}) &= 12(1 + \frac{2\hat{m}_\ell^2}{\hat{s}})
\end{aligned}
\tag{24}
$$

Before turning to a derivation of CP violating asymmetries for the case of polarized final state leptons, it is necessary to remind the calculation of the lepton polarization. The spin direction of a lepton can be described by setting a reference frame with three orthogonal unit vectors S_L, S_N and S_T, such that

$$
\begin{aligned}
S_L &= \frac{p^-}{|p^-|} , \\
S_N &= \frac{p_d \times p^-}{|p_d \times p^-|} , \\
S_T &= S_N \times S_L ,
\end{aligned}
\tag{25}
$$

where p_d and p^- are the three momentum vectors of the d quark and the ℓ^- lepton, respectively, in the $\ell^+\ell^-$ center-of-mass(CM) system. For a given lepton ℓ^- spin direction $\vec{n}$, which is a unit vector in the ℓ^- rest frame, the differential decay spectrum is of the form [33]

$$\frac{d\Gamma(\hat{s},\vec{n})}{d\hat{s}} = \frac{1}{2}\left(\frac{d\Gamma(\hat{s})}{d\hat{s}}\right)_0 \left[1 + (P_L e_L + P_T e_T + P_N e_N) \cdot \vec{n}\right], \tag{26}$$

where the polarization components P_i (i = L, N, T) are obtained from the relation

$$P_i(\hat{s}) = \frac{d\Gamma(\vec{n}=e_i)/d\hat{s} - d\Gamma(\vec{n}=-e_i)/d\hat{s}}{d\Gamma(\vec{n}=e_i)/d\hat{s} + d\Gamma(\vec{n}=-e_i)/d\hat{s}} = \frac{\Delta_i^{\mathcal{U}}(\hat{s})}{\Delta^{\mathcal{U}}(\hat{s})} . \tag{27}$$

The three different polarization asymmetries are:

$$
\begin{aligned}
P_L(\hat{s}) &= \frac{\Delta_L^{\mathcal{U}}(\hat{s})}{\Delta^{\mathcal{U}}(\hat{s})} = \frac{v}{\Delta^{\mathcal{U}}(\hat{s})}\Big[12\mathrm{Re}(C_7^{eff}C_{10}^{\mathcal{U}*}) + 2\mathrm{Re}(C_9^{\mathcal{U}}C_{10}^{\mathcal{U}*})(1 + 2\hat{s})\Big], \\
P_T(\hat{s}) &= \frac{\Delta_T^{\mathcal{U}}(\hat{s})}{\Delta^{\mathcal{U}}(\hat{s})} = \frac{3\pi\hat{m}_\ell}{2\Delta^{\mathcal{U}}(\hat{s})\sqrt{\hat{s}}}\Big[2\mathrm{Re}(C_7^{eff}C_{10}^{\mathcal{U}*}) - 4\mathrm{Re}(C_7^{eff}C_9^{\mathcal{U}*}) \\
&\quad - \frac{4}{\hat{s}}|C_7^{eff}|^2 + \mathrm{Re}(C_9^{\mathcal{U}}C_{10}^{\mathcal{U}*}) - |C_9^{\mathcal{U}}|^2\hat{s}\Big], \\
P_N(\hat{s}) &= \frac{\Delta_N^{\mathcal{U}}(\hat{s})}{\Delta^{\mathcal{U}}(\hat{s})} = \frac{3\pi\hat{m}_\ell v}{2\Delta^{\mathcal{U}}(\hat{s})}\sqrt{\hat{s}}\,\mathrm{Im}(C_9^{\mathcal{U}*}C_{10}^{\mathcal{U}}) \,,
\end{aligned}
\tag{28}
$$

The study of the above-mentioned asymmetries is interesting in probing new physics. As it is obvious that any alteration in the Wilson coefficients leads to changes in the polarization asymmetries.

Now, we define the polarized CP asymmetry which is:

$$
A_{CP}(\hat{s}) = \frac{\frac{d\Gamma(\hat{s},\vec{n})}{d\hat{s}} - \frac{d\bar{\Gamma}(\hat{s},\vec{\bar{n}})}{d\hat{s}}}{(\frac{d\Gamma(\hat{s})}{d\hat{s}})_0 + (\frac{d\bar{\Gamma}(\hat{s})}{d\hat{s}})_0} \,,
\tag{29}
$$

where

$$
\begin{aligned}
\frac{d\Gamma(\hat{s},\vec{n})}{d\hat{s}} &= \frac{d\Gamma(b \to d\ell^+\ell^-(\vec{n}))}{d\hat{s}} \,, \\
\frac{d\bar{\Gamma}(\hat{s},\vec{\bar{n}})}{d\hat{s}} &= \frac{d\Gamma(\bar{b} \to \bar{d}\ell^+(\vec{\bar{n}})\ell^-)}{d\hat{s}} \,,
\end{aligned}
\tag{30}
$$

here, $\vec{n}$ and $\vec{\bar{n}}$ are the spin directions for ℓ^- and ℓ^+ for b-decay and $\bar{b}$-decay, respectively, and $i = L,\ N,\ T$. Taking into account the fact that $\vec{\bar{e}}_{L,N} = -\vec{e}_{L,N}$, and $\vec{\bar{e}}_T = \vec{e}_T$, we obtain

$$
A_{CP}(\vec{n} = \pm\vec{e}_i) = \frac{1}{2}\left\{ \frac{(\frac{d\Gamma}{d\hat{s}})_0 - (\frac{d\bar{\Gamma}}{d\hat{s}})_0}{(\frac{d\Gamma}{d\hat{s}})_0 + (\frac{d\bar{\Gamma}}{d\hat{s}})_0} \pm \frac{(\frac{d\Gamma}{d\hat{s}})_0 P_i - \left((\frac{d\Gamma}{d\hat{s}})_0 P_i\right)_{|\lambda_u \to \lambda_u^*}}{(\frac{d\Gamma}{d\hat{s}})_0 + (\frac{d\bar{\Gamma}}{d\hat{s}})_0} \right\} \,.
\tag{31}
$$

Using Eq. (28), we get from Eq. (31),

$$
\begin{aligned}
A_{CP}(\vec{n} = \pm\vec{e}_i) &\approx \frac{1}{2}\left\{ \frac{\Delta^{\mathcal{U}} - \bar{\Delta}^{\mathcal{U}}}{\Delta^{\mathcal{U}} + \bar{\Delta}^{\mathcal{U}}} \pm \frac{\Delta_i^{\mathcal{U}} - \bar{\Delta}_i^{\mathcal{U}}}{\Delta^{\mathcal{U}} + \bar{\Delta}^{\mathcal{U}}} \right\} \,, \\
&= \frac{1}{2}\left\{ A_{CP}(\hat{s}) \pm A_{CP}^i(\hat{s}) \right\} \,,
\end{aligned}
\tag{32}
$$

where the upper sign in the definition of δA_{CP} corresponds to L and N polarizations, while the lower sign corresponds to T polarization.

The $A_{CP}^i(\hat{s})$ terms in Eq. (32) describe the modification to the unpolarized decay width, which can be written as:

$$A_{CP}^i(\hat{s}) = \frac{-4\text{Im}(\lambda_u)\Sigma^i(\hat{s})}{\Delta^{\mathcal{U}}(\hat{s}) + \bar{\Delta}^{\mathcal{U}}(\hat{s})} ,$$

$$\approx -2\text{Im}(\lambda_u)\frac{\Sigma^i(\hat{s})}{\Delta^{\mathcal{U}}(\hat{s})} . \tag{33}$$

where, the explicit expressions for $\Sigma^i(\hat{s}), \ (i = L, N, T)$, are as follows:

$$
\begin{aligned}
\Sigma^L(\hat{s}) &= v\text{Im}(C_{10}^{\mathcal{U}*}\xi_2)(1 + 2\hat{s}) \\
\Sigma^T(\hat{s}) &= \frac{3\pi\hat{m}_\ell}{2\sqrt{\hat{s}}}\left[2\text{Im}(C_7^{\text{eff}}\xi_2^*) + \frac{1}{2}\text{Im}(C_{10}^{\mathcal{U}*}\xi_2) - \hat{s}\text{Im}(\xi_1^{\mathcal{U}*}\xi_2)\right] \\
\Sigma^N(\hat{s}) &= \frac{3\pi\hat{m}_\ell}{2\sqrt{\hat{s}}}v\left[\frac{\hat{s}}{2}\text{Re}(C_{10}^{\mathcal{U}*}\xi_2)\right]
\end{aligned}
\tag{34}
$$

It is interesting to note that the polarized CP asymmetries have different combinations involving the imaginary and real parts of the $C_{10}^{\mathcal{U}}$ which doesn't appear in unpolarized CP asymmetry. The study of the polarized CP asymmetry beside the unpolarized CP asymmetry with unparticle contributions will give us more information about the unparticle parameters. In particular, when $C_L^\ell = -C_R^\ell$ in (14), the unparticle contribution vanishes in the $C_9^{\mathcal{U}}$. In such situation, the unparticle effects in CP asymmetry just appear in the polarized CP asymmetries.

4 Numerical Analysis and Discussion

We try to analyze the dependency of the unpolarized and polarized direct CP asymmetries on the unparticle parameters . We will use the next–to–leading order logarithmic approximation for the SM values of the Wilson coefficients C_9^{eff}, C_7^{eff} and C_{10}^{eff} Ref. [34, 35] at the scale $\mu = m_b$. It is worth to mention that, beside the short distance contribution, C_9^{eff} has also long distance contributions resulting from the real $\bar{c}c$ resonant states of the J/ψ family. In the present study, we do not take the long distance effects into account. Furthermore, one finds that significant contributions of unparticle occurs at small region of $\hat{s}$ which is free of long distance effects(obviously, the unparticle contributions for μ channel is significant than the τ channel since small $\hat{s}$ region($\hat{s} \sim 0.0$) for τ channel is absent by kinematical consideration). One can confirm the above statement by looking at Eqs. (2) and (5) where at the small $\hat{s} = q^2/m_b^2$ region the dependency of the propagator is as follows:

$$\left[\frac{1}{q^2}\left(\frac{q^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}-1}\right]^2 . \tag{35}$$

Parameter	Value
α_{em}	1/129 (GeV)
m_u	2.3 (MeV)
m_d	4.6 (MeV)
m_c	1.25 (GeV)
m_b	4.8 (GeV)
m_μ	0.106 (GeV)
m_τ	1.780 (GeV)

Table 1: The values of the input parameters used in the numerical calculations.

The SM parameters we used in this analysis are shown in Table 1:

The allowed range for the Wolfenstein parameters are: $0.19 \leq \rho \leq 0.268$ and $0.305 \leq \eta \leq 0.411$ [36] where, in the present analysis they are set as $\rho = 0.25$ and $\eta = 0.34$.

The direct CP asymmetries depend on both $\hat{s}$ and the new parameters coming from unparticle stuff. We eliminate the variable $\hat{s}$ by performing integration over $\hat{s}$ in the allowed kinematical region. The averaged direct CP asymmetries are defined as:

$$
\begin{aligned}
\mathcal{B}_r &= \int_{4m_\ell^2/m_b^2}^{(1-\sqrt{\hat{r}_d})^2} \frac{d\mathcal{B}}{d\hat{s}} d\hat{s}, \quad (\hat{r}_d = \frac{m_d^2}{m_b^2}) \\
\left\langle A_{CP}^i \right\rangle &= \frac{\int_{4m_\ell^2/m_b^2}^{(1-\sqrt{\hat{r}_d})^2} A_{CP}^i \frac{d\mathcal{B}}{d\hat{s}} d\hat{s}}{\mathcal{B}_r} .
\end{aligned}
\tag{36}
$$

At this stage, we discuss our restrictions for free parameters coming out of the unparticle:

- It is important to note that while the discontinuity across the cut is not singular for integer $d_{\mathcal{U}} > 1$, the propagator (Eq. 6) is singular because of the $sin(d_{\mathcal{U}}\pi)$ in the denominator. Some researchers believe that this is a real effect[2]. These integer values describe multiparticle cuts and the mathematics tells us that we should not try to describe them with a single unparticle field.

  Moreover, the lower bounds for the scaling dimensions of the gauge invariant vector operators of a CFT are $d_{\mathcal{U}} \geq 2$ and $d_{\mathcal{U}} \geq 3$[12] for non-primary and primary vector operators, respectively. We obtain that for $d_{\mathcal{U}} > 2$ the unparticle effects on physical observables(branching ratio, CP asymmetry and so on) almost vanish because $\tilde{\Delta}(p^2)$ is negligible for $p < \Lambda_{\mathcal{U}}$(see Eq. 9).

  We focus on $1 < d_{\mathcal{U}} < 2$, the bound that is allowed for transverse $O_{\mathcal{U}}^\mu$ or for non-gauge invariant vector operators of the CFT. Also, it is consistent with the $b \to s\ell^+\ell^-$ rate[8] and B_s mixing[9]. We also assume that the virtual effect of unparticles are gentlest away

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from the integer boundaries. On the other hand, the momentum integrals converges for $d_\mathcal{U} < 2$ [13].

- C_L^t is always associated with C_R^ℓ and C_R^ℓ(see Eq. (14). For simplicity, we set $C_R^\ell = C_L^\ell$ or $C_R^\ell = -C_L^\ell$. We will set new parameters to be $C_L^t C_L^\ell = C_L^t C_R^\ell = \lambda_V^\ell$ and $C_L^t C_L^\ell = -C_L^t C_R^\ell = \lambda_A^\ell$ and choose the $\lambda_{V[A]}^\ell = 0.005$, 0.01 and 0.05 which is consistent with the $b \to s\ell^+\ell^-$ rate[8].

- We take the energy scale $\Lambda_\mathcal{U} = 1\,(TeV)$ and study $d_\mathcal{U}$ dependence of the polarized and unpolarized CP asymmetry.

CP asymmetry is good candidate(unlike the other physical observables i.e,. branching ratio, forward-backward asymmetry and ...) to probe the unique unparticle phase. The other physical observables can utilize to give strong constraints on the unparticle parameters except the phase, *i.e.* on the unparticle couplings to leptons such as $\lambda_{V(A)}^\ell \sim \{0.005 - 0.05\}$[8]. Moreover, our numerical analysis confirm the result of the [8] where the branching ratio(BR) of $b \to s(d)\ell^+\ell^-$ decay depict the strong enhancement at the low value of the scale dimension $d_\mathcal{U} \sim 1.1$ with respect the SM value. As a natural consequence of this feature, the averaged value of asymmetries will vanish unless they depict stronger enhancement than the BR.

The contributions of unparticle to the CPA of $b \to d\ell^+\ell^-$ in terms of the values for the common parameters are presented in Fig. 3–10. The horizontal thin lines are the SM contributions, the dashed lines and dash–pointed lines correspond to the different $\lambda_{A[V]}^\ell = 0.005$, 0.01 and 0.05 , respectively. From theses figures, we conclude that:

- $\langle A_{CP} \rangle$ for both μ and τ leptons depicts strong dependency on the unparticle effects(for μ case the dependency is stronger that τ case as we discussed above). While it is suppressed to the zero value by the unparticle contributions at lower values of the scale dimension $d_\mathcal{U} \sim 1.1$. Its value is close to the SM value at the higher values of the scale dimension $d_\mathcal{U} \sim 1.9$. Moreover, the sensitivity for different values of the λ_A is stronger and more interesting than the λ_V values. While for different λ_V values, $\langle A_{CP} \rangle$ is just decreasing in terms of the $d_\mathcal{U}$, but for λ_A it is increasing, decreasing and changing the sign(see figs. 3, and 4).

- $\langle A_{CP}^L \rangle$ for both μ and τ leptons shows strong dependency on the unparticle parameters. While it is suppressed to the zero value by the unparticle contributions at lower values of the scale dimension $d_\mathcal{U} \sim 1.1$(see figs. 5, and 6), its value is close to the SM value

at the higher values of the scale dimension $d_{\mathcal{U}} \sim 1.9$. The situation for the μ leptons is much more interesting. While the SM value is about few percent, but it receive sizable and measurable contribution up to 10% from unparticle effects(see figs. 5). As $\langle A^L_{CP} \rangle$ and $\langle A_{CP} \rangle$ are sensitives to the $C^{\mathcal{U}}_{10}$ and $C^{\mathcal{U}}_9$, respectively, thus, the study of $\langle A^L_{CP} \rangle$ beside $\langle A_{CP} \rangle$ is supplementary and complementary to study unparticle effects. More precisely, unlike $\langle A_{CP} \rangle$, $\langle A^L_{CP} \rangle$ shows stronger dependency on the the different values of the λ_A than the λ_V values.

- $\langle A^T_{CP} \rangle$ is generally sensitive to the unparticle contributions for both μ and τ channels. While, the SM values of $\langle A^T_{CP} \rangle$ almost vanishes, the unparticle contributions lead to sizable deviation from the SM values,(see figs. 7 and 8). This sizable discrepancy from the SM values can be measured in future experiments like LHC and ILC.

- Either the SM value or its value with unparticle contributions for $\langle A^N_{CP} \rangle$ is negligible(see figs. 9 and 10).

At the end, the quantitative estimation about the accessibility to measure the various physical observables are in order. An observation of a 3σ signal for CP asymmetry of the order of the 1% requires about $\sim 10^{10}$ $B\bar{B}$ pair[33]. For $b \to d\ell^+\ell^-$ measurement a good d-quark tagging is necessary to distinguish it from much more stronger $b \to s\ell^+\ell^-$ decay signal. Putting aside this challenging task, the number of $B\bar{B}$ pairs, expected to produce at LHC, are about $\sim 10^{12}$. As a result of comparison of these values, we conclude that a typical asymmetry of $(\mathcal{A} = 1\%)$ is certainly detectable at LHC.

In conclusion, firstly, we obtain that the unparticle effects on physical observables i.e., branching ratio and CP asymmetry for $b \to d(s)\ell^+\ell^-$ decays when $d_{\mathcal{U}} \geq 2$ vanish. Secondly, for $1 < d_{\mathcal{U}} < 2$, the CP asymmetry for polarized and unpolarized lepton cases are studied within the unparticle contributions in the CPA of the $b \to d\ell^+\ell^-$ decays. We obtain that the unpolarized and polarized CP asymmetries are strongly sensitive to the unparticle effects. In particular, the CPA for small values of scale dimension $d_{\mathcal{U}} \sim 1.1$ suppresses to zero and for its definite values the CPA enhances considerably and changes its sign with respect to the corresponding SM value. The other parameters of the scenario studied are the $\mathcal{U}$-fermion-fermion couplings, the energy scale and the dependencies of the CPA to these free parameters are also strong. We show that a measurement of the magnitude and sign of the unpolarized and polarized asymmetries can be instructive in order to test the possible signals coming from the unparticle physics.

5 Acknowledgment

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Figure captions

Fig. (3) The dependence of the $\langle \mathcal{A}_{CP} \rangle$ for the $b \to d\mu^+\mu^-$ decay on $d_{\mathcal{U}}$ for three different values of $\lambda_V : 0.005,\ 0.01,\ 0.05$ and $\lambda_A : 0.005,\ 0.01,\ 0.05$ in the fixed value of $\Lambda_{\mathcal{U}} = 1\text{TeV}$.

Fig. (4) The same as in Fig. (1), but for the τ lepton.

Fig. (5) The dependence of the $\langle \mathcal{A}_{CP}^L \rangle$ for the $b \to d\mu^+\mu^-$ decay on $d_{\mathcal{U}}$ for three different values of $\lambda_V : 0.005,\ 0.01,\ 0.05$ and $\lambda_A : 0.005,\ 0.01,\ 0.05$ in the fixed value of $\Lambda_{\mathcal{U}} = 1\text{TeV}$.

**Fig. (6)** The same as in Fig. (3), but for the τ lepton.

Fig. (7) The dependence of the $\langle \mathcal{A}_{CP}^T \rangle$ for the $b \to d\mu^+\mu^-$ decay on $d_{\mathcal{U}}$ for three different values of $\lambda_V : 0.005,\ 0.01,\ 0.05$ and $\lambda_A : 0.005,\ 0.01,\ 0.05$ in the fixed value of $\Lambda_{\mathcal{U}} = 1\text{TeV}$.

Fig. (8) The same as in Fig. (5), but for the τ lepton .

Fig. (9) The dependence of the $\langle \mathcal{A}_{CP}^N \rangle$ for the $b \to d\mu^+\mu^-$ decay on $d_{\mathcal{U}}$ for three different values of $\lambda_V : 0.005,\ 0.01,\ 0.05$ and $\lambda_A : 0.005,\ 0.01,\ 0.05$ in the fixed value of $\Lambda_{\mathcal{U}} = 1\text{TeV}$.

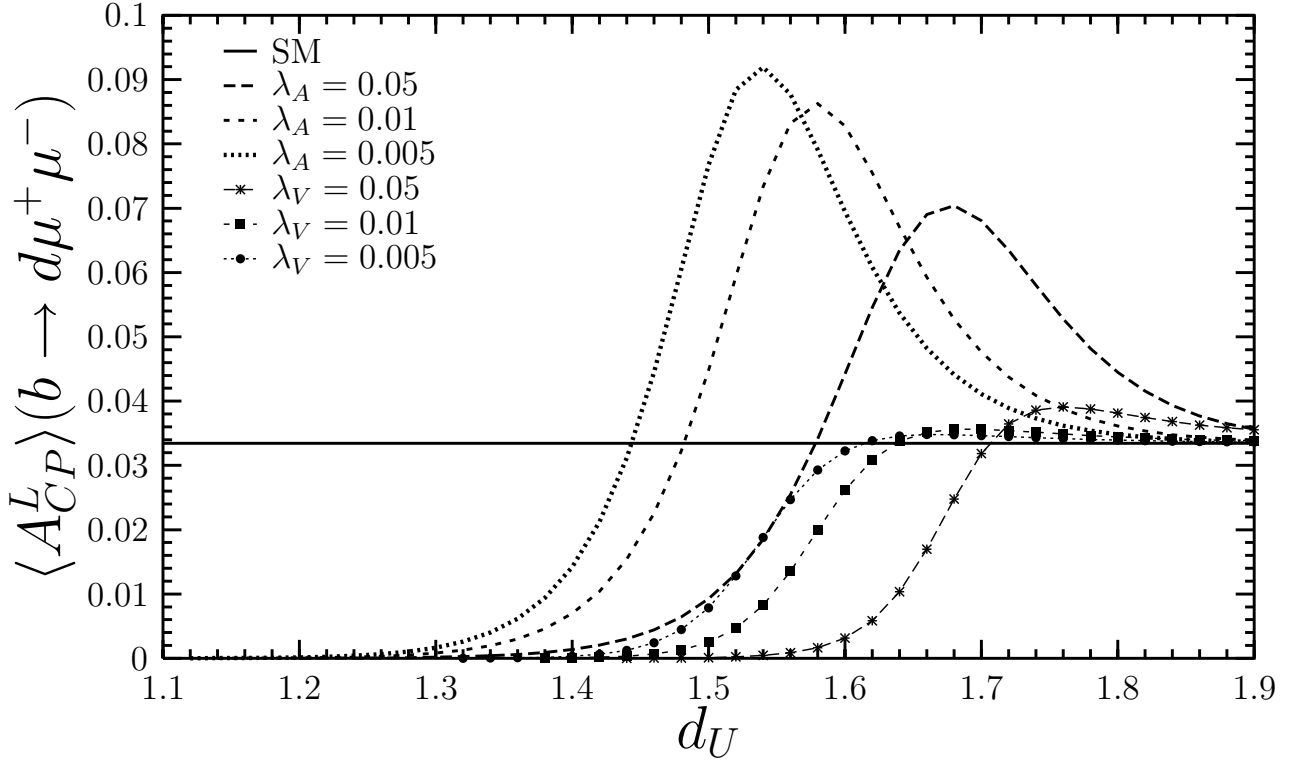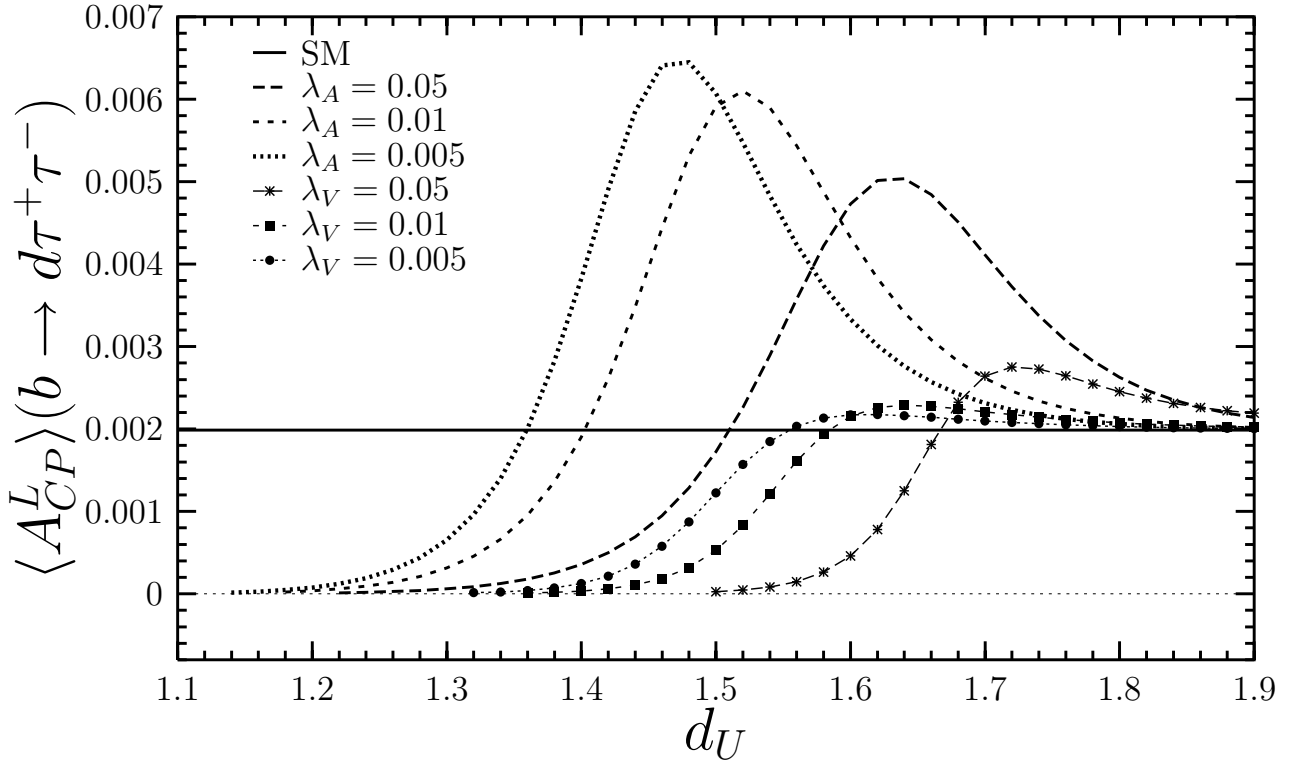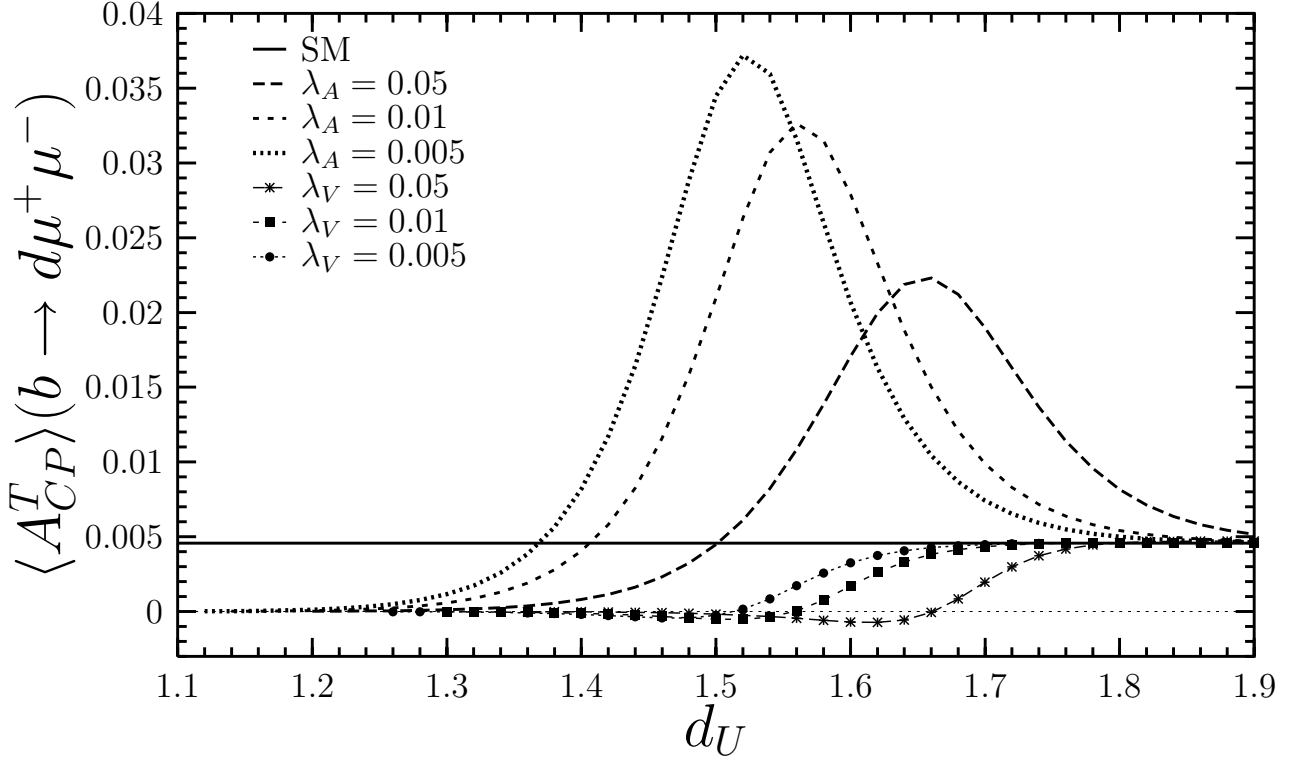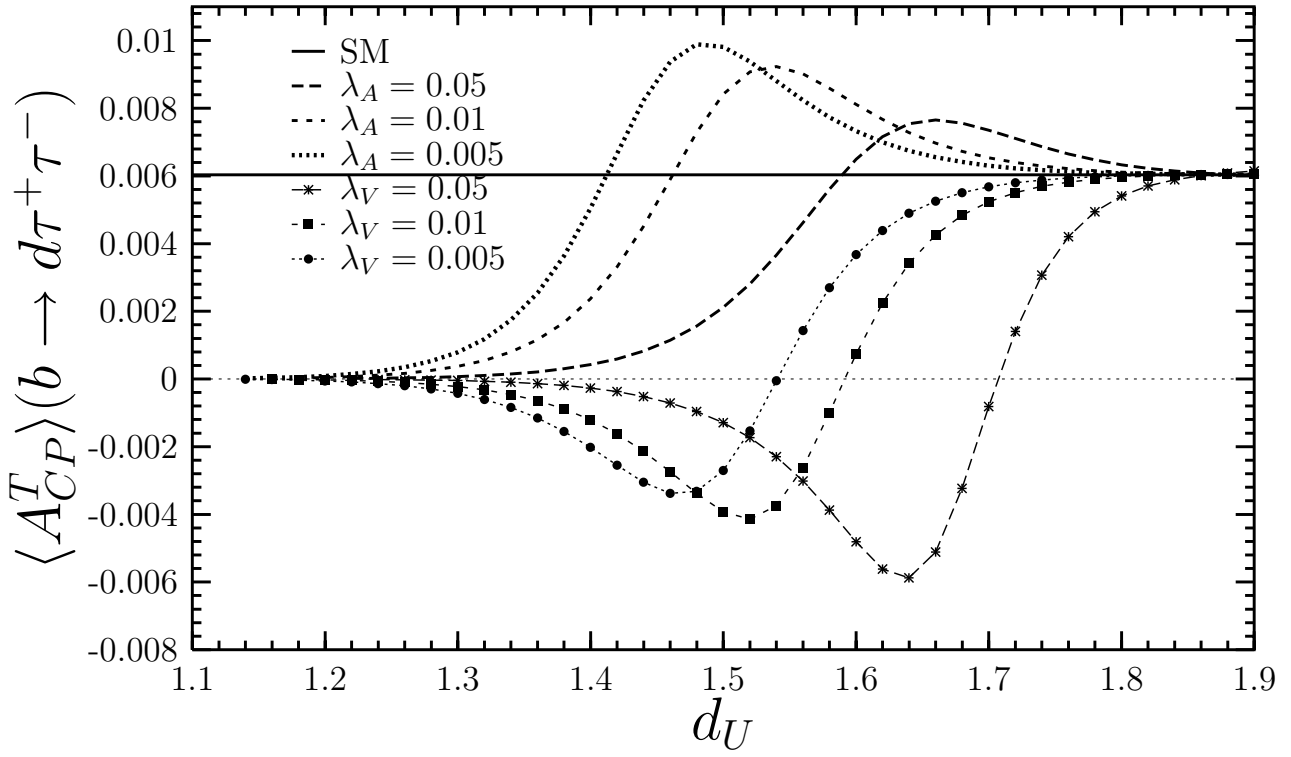**Fig. (10)** The same as in Fig. (5), but for the τ lepton.

Figure 3:

Figure 4:

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Figure 5:

Figure 6:

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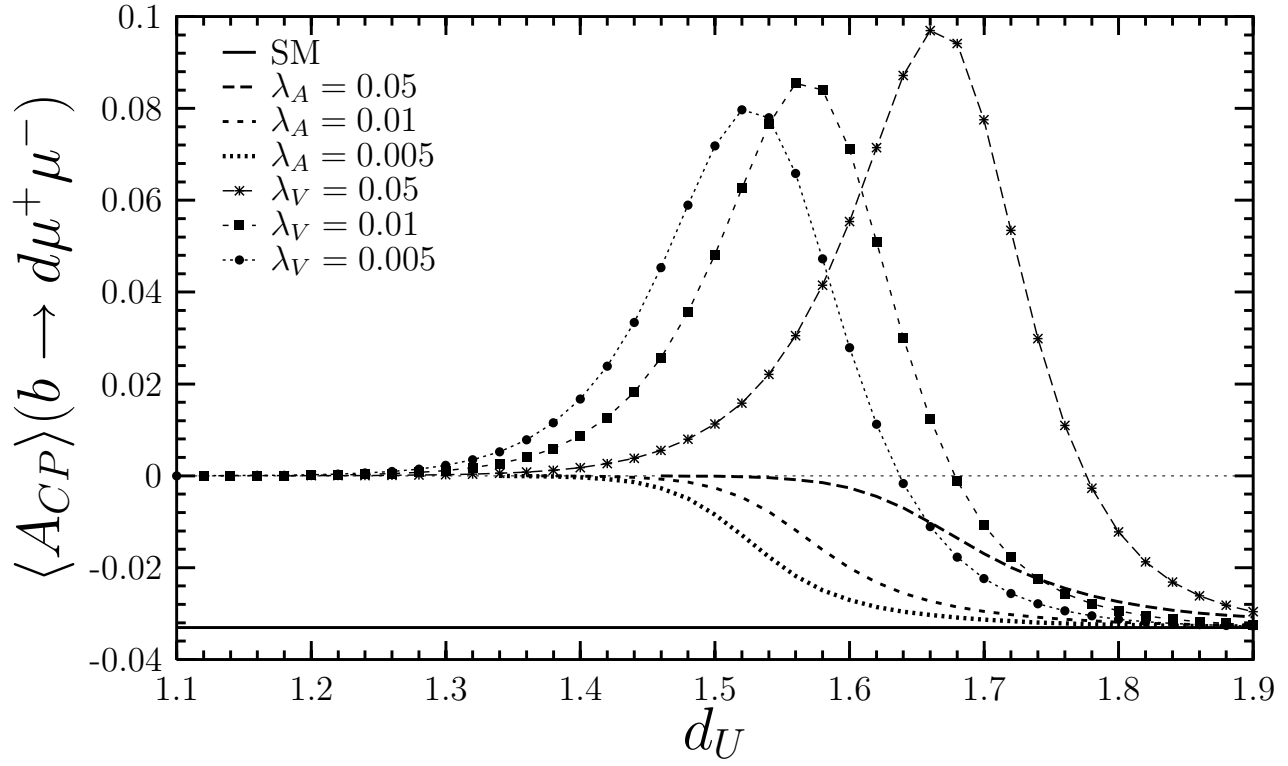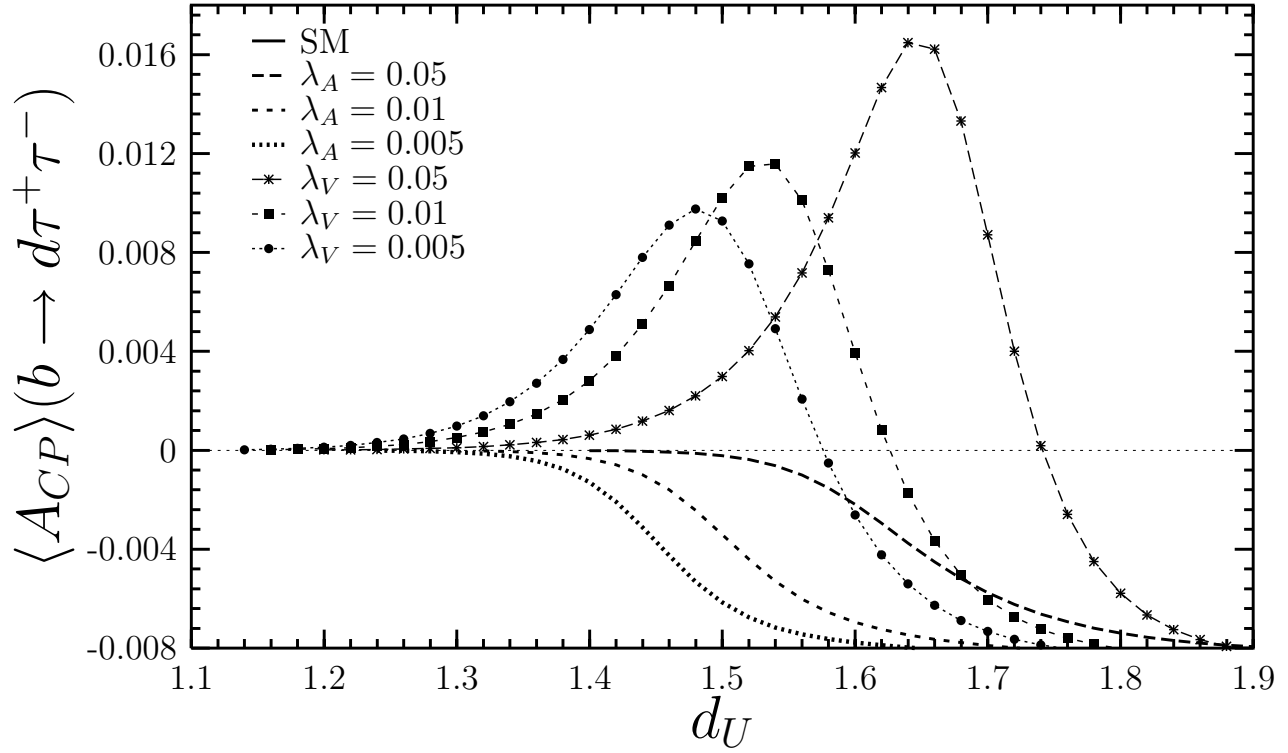
Figure 7:

Figure 8:

Figure 9:

Figure 10: